Advanced Math 2 HONORS
REVIEW
Unit 6 Lessons 1-2

Hi! My name is Key and today is ________.

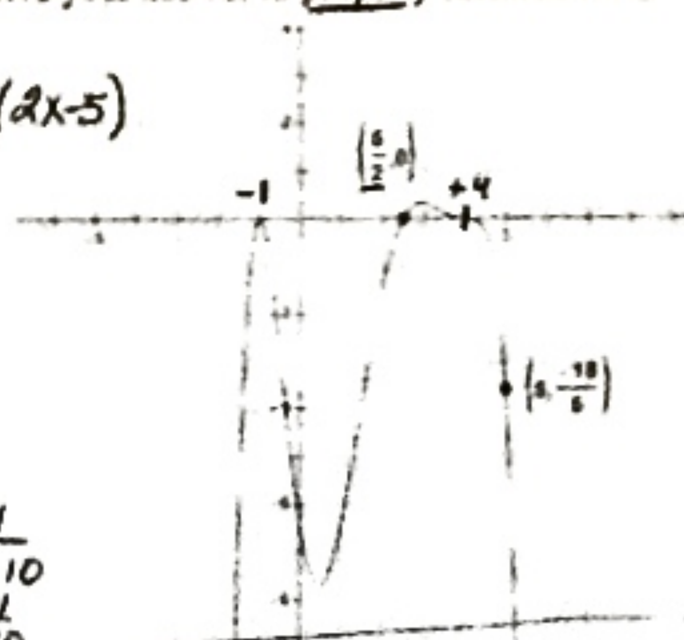
1. Find the equation of the polynomial that has 3 distinct zeros, each with multiplicity of one, at $x = -4$, 1, and 5 and passes through the point (0,10). Show your work.

$y = a(x+4)(x-1)(x-5)$
$10 = a(0+4)(0-1)(0-5)$
$10 = a(4)(-1)(-5)$
$10 = a(20)$
$\frac{10}{20} = a$

$\boxed{y = \frac{1}{2}(x+4)(x-1)(x-5)}$

2. Find the equation of the polynomial function below. Give your answer in properly factored form

roots at $-1$ (mult. 2) $\rightarrow (x+1)^2$
$\frac{5}{2}$ (mult. 1) $\rightarrow (x-\frac{5}{2}) \rightarrow (2x-5)$
$4$ (mult. 3) $\rightarrow (x-4)^3$



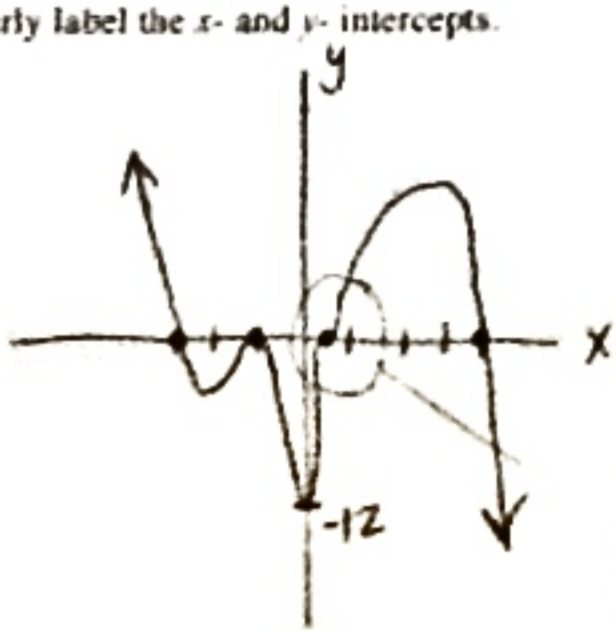

$y = a(x+1)^2(2x-5)(x-4)^3$
$-\frac{18}{5} = a(5+1)^2(2\cdot 5-5)(5-4)^3$
$-\frac{18}{5} = a(6)^2(5)(1)^3$
$-\frac{18}{5} = a\, 36\cdot 5$
$\frac{-18}{5\cdot 36\cdot 5} = a$ $\rightarrow$ $a = \frac{-18}{36\cdot 10} = -\frac{1}{2\cdot 10}$
$a = -\frac{1}{20}$

$\boxed{y = -\frac{1}{20}(x+1)^2(2x-5)(x-4)^3}$

3. Create a function that has different zeros with different multiplicities and has a degree of 9.

(many correct answers) e.g. $(x-1)^2(x-2)^3(x-4)^4$

4. Sketch a graph of the following equation: $g(x) = -(x+3)(x+1)^2(2x-1)^3(x-4)$.
Clearly label the $x$- and $y$- intercepts.




roots $-3$ $-1$ $\frac{1}{2}$ $4$
mult 2 mult 3

degree 7
- lead coef

↖ ↘

y-int
$-(0+3)(0+1)^2(2\cdot 0-1)^3(0-4)$
$-(3)(1)^2(-1)^3(-4)$
$-3\cdot 1\cdot -1\cdot -4$
$-12$
$(0,-12)$

Advanced Math 2 HONORS
REVIEW
Unit 6 Lessons 1-2

~~1~~. Find the equation of the polynomial passing through these points: (-3, 92), (-1, 0), (0, -4), and (3, -64). Show or explain how you got the equation.

4 pts → degree 3

so $y = ax^3 + bx^2 + cx + d$

$$\left.\begin{aligned} 92 &= a(-3)^3 + b(-3)^2 + c(-3) + d \\ 0 &= a(-1)^3 + b(-1)^2 + c(-1) + d \\ -4 &= a(0)^3 + b(0)^2 + c(0) + d \\ -64 &= a(3)^3 + b(3)^2 + c(3) + d \end{aligned}\right\} \begin{aligned} 92 &= -27a + 9b - 3c + d \\ 0 &= -a + b - c + d \\ -4 &= d \\ -64 &= 27a + 9b + 3c + d \end{aligned}$$

$\underline{d = -4}$

so
$$\begin{aligned} 96 &= -27a + 9b - 3c \\ \underline{4} &= \underline{-a + b - c} \\ -60 &= 27a + 9b + 3c \end{aligned}$$

→ $\underline{c = -a + b - 4}$

$96 = -27a + 9b - 3(-a + b - 4)$ → $96 = \underline{-27a} + \underline{9b} + \underline{3a} - \underline{3b} + 12$

$-60 = 27a + 9b + 3(-a + b - 4)$ → $-60 = \underline{27a} + \underline{9b} - \underline{3a} + \underline{3b} - 12$

↓

$96 = -24a + 6b + 12$
$-60 = 24a + 12b - 12$

↓

$$\begin{aligned} 84 &= -24a + 6b \\ + \quad -48 &= 24a + 12b \\ \hline 36 &= 18b \\ \underline{2} &\underline{= b} \end{aligned}$$

$84 = -24a + 6(2)$
$84 = -24a + 12$
$72 = -24a$
$\underline{-3 = a}$

$c = -a + b - 4$
$c = --3 + 2 - 4$
$\underline{c = 1}$

so $\boxed{y = -3x^3 + 2x^2 + x - 4}$

Advanced Math 2 HONORS
REVIEW
Unit 6 Lessons 1-2

6. (a.) Use **long division** to find the quotient when $2x^4-22x^3+66x^2-10x-100$ is divided by $(2x-10)$.

$$\boxed{x^3-6x^2+3x+10}$$

$$2x-10\,\overline{)\,2x^4-22x^3+66x^2-10x-100}$$

$$-2x^4+10x^3$$

$$-12x^3+66x^2$$

$$+12x^3\mp 60x^2$$

$$6x^2-10x$$

$$-6x^2+30x$$

$$20x-100$$

$$-20x+100 \quad 0$$

(b.) Using your work in part (a.), write $2x^4-22x^3+66x^2-10x-100$ as the **product** of a linear factor and a cubic factor.

$$\boxed{(2x-10)(x^3-6x^2+3x+10)}$$

(c.) Write $2x^4-22x^3+66x^2-10x-100$ in fully factored form.

$$x^3-6x^2+3x+10=(x+1)(x^2-7x+10)=(x+1)(x-5)(x-2)$$

PRZ's
$\pm 1, 2, 5, 10$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 3 & 10 \\ & & 1 & -5 & -2 \\ \hline & 1 & -5 & -2 & \text{X} \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 3 & 10 \\ & & -1 & 7 & -10 \\ \hline & 1 & -7 & 10 & 0 \end{array}$$

so $2x^4-22x^3+66x^2-10x-100=$
$(2x-10)(x+1)(x-5)(x-2)$

$$\boxed{2(x-5)(x+1)(x-5)(x-2)}$$

7. Factor $g(x)=-x^3+3x^2-2x+6$ into the polynomials of the smallest possible degrees.

$$g(x)=-(x^3-3x^2+2x-6)$$

PRZ's
$\pm 1, 2, 3, 6$

$$\boxed{g(x)=-(x-3)(x^2+2)}$$

can't be factored w/ integers

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 2 & -6 \\ & & 1 & -2 & 0 \\ \hline & 1 & -2 & 0 & -6\ \text{X} \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & -3 & 2 & -6 \\ & & 2 & -2 & 0 \\ \hline & 1 & -1 & 0 & -6\ \text{X} \end{array} \qquad \begin{array}{r|rrrr} 3 & 1 & -3 & 2 & -6 \\ & & 3 & 0 & 6 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 2 & -6 \\ & & -1 & 4 & -6 \\ \hline & 1 & -4 & 6 & -12\ \text{X} \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & -3 & 2 & -6 \\ & & -2 & 10 & -24 \\ \hline & 1 & -5 & 12 & \text{X} \end{array}$$

# Advanced Math 2 HONORS
## REVIEW
## Unit 6 Lessons 1-2

~~8~~. What are the solutions to $8 = x^2 - 5x - 6$?

$$0 = x^2 - 5x - 14$$
$$0 = (x-7)(x+2)$$
$$x-7=0 \quad x+2=0$$
$$\boxed{x=7,\ x=-2}$$

~~9~~. Solve the following equation: $0 = x^4 + 3x^3 - 3x^2 - 11x - 6$

PRZ$^s$
$\pm 1, 2, 3, 6$

$$\begin{array}{r|rrrrr} & 1 & 3 & -3 & -11 & -6 \\ 1 & & 1 & 4 & 1 & -10 \\ \hline & 1 & 4 & 1 & -10 & -16 \end{array} \; \times$$

$$\begin{array}{r|rrrrr} & 1 & 3 & -3 & -11 & -6 \\ -1 & & -1 & -2 & 5 & 6 \\ \hline & 1 & 2 & -5 & -6 & 0 \end{array}$$

$$\downarrow$$

$$(x+1)(x^3 + 2x^2 - 5x - 6)$$

PRZ$^s$
$\pm 1, 2, 3, 6$

$$\begin{array}{r|rrrr} & 1 & 2 & -5 & -6 \\ 2 & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$$\downarrow$$

$$(x+1)(x-2)(x^2 + 4x + 3)$$
$$(x+1)(x-2)(x+3)(x+1) = 0$$
so $\boxed{x = -1, 2, -3}$

~~10~~. What are the complex solutions to $2x^2 - 2x - 3 = -5$?

$$2x^2 - 2x + 2 = 0$$
$$2(x^2 - x + 1) = 0$$
so $x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{1-4}}{2}$$
$$x = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2} = \boxed{\frac{1}{2} \pm \frac{\sqrt{3}}{2} i}$$

11. Consider the following polynomial function: $p(x) = x^3 - 2x^2 + 9x - 18$
Find all complex roots of this polynomial. Use polynomial long division and the quadratic formula to help you.

PRZ^s
$\pm 1, 2, 3, 6, 9, 18$

Test..
$x=2$ works

| | | | | |
|---|---|---|---|---|
| | 1 | -2 | 9 | -18 |
| 2 | | 2 | 0 | 18 |
| | 1 | 0 | 9 | 0 |

so $p(x) = (x-2)(x^2+9)$

$x - 2 = 0$
$\boxed{x = 2}$

$x^2 + 9 = 0$
$x^2 = -9$
$x = \pm\sqrt{-9}$
$\boxed{x = \pm 3i}$

12. Verify that $x=1$ is a root of $f(x) = x^4 + 3x^3 - 8x^2 + 4$.

either
$f(1) = 1^4 + 3(1)^3 - 8(1)^2 + 4$
$1 + 3\cdot 1 - 8\cdot 1 + 4$
$1 + 3 - 8 + 4$
$4 - 8 + 4 = 0$ ✓

OR

| | | | | |
|---|---|---|---|---|
| | 1 | 3 | -8 | 4 |
| 1 | | 1 | 4 | -4 |
| | 1 | 4 | -4 | 0 |

R=0 ✓

13. Verify that $x = -1 + \sqrt{3}i$ is a root of $y = x^3 - 8$.

$y = (-1+\sqrt{3}i)^3 - 8$
$= (-1+\sqrt{3}i)(-1+\sqrt{3}i)(-1+\sqrt{3}i) - 8$
$= (1 - \sqrt{3}i - \sqrt{3}i + 3i^2)(-1+\sqrt{3}i) - 8$
$= (1 - 2\sqrt{3}i - 3)(-1+\sqrt{3}i) - 8$
$= (-2 - 2\sqrt{3}i)(-1+\sqrt{3}i) - 8$
$= 2 - 2\sqrt{3}i + 2\sqrt{3}i - 2\cdot 3i^2 - 8$
$= 2 + 6 - 8 = 0$ ✓

Advanced Math 2 HONORS
REVIEW
Unit 6 Lessons 1-2

14. Perform the indicated operation. Write your answer in standard form ($a+bi$). Show work.

a. $(3+6i)+(4+9i)$

$\boxed{7+15i}$

b. $(4+2i)-(3+7i)$

$\boxed{1-5i}$

c. $(1-i)-(3-5i)$

$\boxed{-2+4i}$

d. $(2+3i)(7-2i)$

$14-4i+21i-6i^2$

$14+17i+6$

$\boxed{20+17i}$

e. $(2+2i)^2$

$(2+2i)(2+2i)$

$4+4i+4i+4i^2$

$4+8i+4i^2$

$4+8i-4$

$\boxed{8i}$

f. $\frac{4+2i}{3-3i}\cdot\frac{3+3i}{3+3i}$

$\frac{12+12i+6i+6i^2}{9-9i^2}$

$\frac{12+18i-6}{9+9}$

$\frac{6+18i}{18}=\boxed{\frac{1}{3}+i}$

g. $\frac{6+5i}{4i}\cdot\frac{-4i}{-4i}$

$\frac{-24i-20i^2}{-16i^2}$

$\frac{-24i+20}{16}$

$\boxed{\frac{5}{4}-\frac{3}{2}i}$

h. What is the complex conjugate of $2+3i$?

$\boxed{2-3i}$